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Static and slowly rotating neutron stars in R^2 gravity

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Abstract. We study non-perturbatively and self-consistently neutron and strange stars in $f(R)$ gravity with Lagrangian $f(R) = R + aR^2$, or the so-called R^2 gravity. Static and slowly rotating models of neutron and strange stars, as well as f-mode oscillations are thoroughly investigated. The results obtained in the R^2 gravity case are compared to the General-relativistic ones. In the mass of radius relations the deviations from GR are about 10%, and in the moment of inertia they are up to 40%. For the asteroseismology relations a high degree of equation of state independence is observed. To some extend insensitivity to the gravitational theory is also observed.

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1 Introduction

The confirmed accelerated expansion of the universe can not be explained in the context of General relativity (GR) without the introduction of new exotic. This is the so-called dark matter which constitutes about 73% of the total energy content of the Universe and exhibits some exotic properties like negative pressure-to-density ratio. As an alternative one can introduce theories alternative to GR in the framework of which the accelerated expansion could be explained without the necessity of exotic matter. In the literature, one of the most popular such class of theories are the so-called $f(R)$ theories where the General-relativistic Lagrangian R is replaced with a more general one, namely a function of R, hence $f(R)$ theories. Any viable cosmological theory should be tested on astrophysical scale as well. In our studies we adopt $f(R) = R+aR^2$, or the so-called R^2 gravity, and study its astrophysical manifestations.

We start with the non-perturbative and self-consistent study of neutron and strange star models in R^2 gravity. Static and slowly rotating models are studied and the study of the latter ones is in slow rotation approximation. The study continues with a major astrophysical implication, namely the gravitational waves asteroseismology. The slow rotation approximation is expected to be valid for rotational frequencies below a few hundred Hz, but most of the observed neutron stars fall in this category. The effect of rotation on the mass and the radius is of second order in the angular velocity Ω of the star, but the slow rotation approximation is of first order in Ω. Therefore, it can not account for the changes of the mass and the radius of the star due to the rotation. However, it can give us information about such global characteristic like the moment of inertia of the neutron star. As an extension to this study we investigate neutron star oscillations, and more precisely the non-radial oscillations which can be a source of gravitational waves. Due to the more complicated equations the study is conducted in the Cowling approximation but the results are qualitatively the same as the results obtained by solving the full problem and the approximation is accurate enough for quantitative study. The latest gravitational waves detections have revealed a new research possibilities for astrophysics. In this context several different asteroseismological relations between the neutron stars parameters are investigated for a wide range of hadronic and quark equations of state (EOS) and their EOS dependence is examined.

2 Basic equations

2.1 Field equations

In this section we briefly present the basic equations describing slowly rotating equilibrium neutron star solutions in R^2 gravity. More details on this problem can be found in [1,2]. The mathematical equivalence between $f(R)$ theories and the scalar-tensor theories (STT) will be employed in this study. For mathematical simplicity it is useful to study the STT in the so-called Einstein frame instead of the physical Jordan frame. More about the transformations between the two frames one can find in [1, 2].

The line element in a stationary and axisymmetric spacetime can be written in the form:

$$ds_*^2 = -e^{2\phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\vartheta^2) - 2\omega(r,\theta)r^2\sin^2\theta d\vartheta dt, \quad (1)$$

where only the first order terms in the angular velocity are kept. The explicit form of the field equations in the Einstein frame is

$$\frac{1}{r^2}\frac{d}{dr}\left[r(1-e^{-2\Lambda})\right] = 8\pi G A^4(\varphi)\rho + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 + \frac{1}{2}V(\varphi), \tag{2}$$

$$\frac{2}{r}e^{-2\Lambda}\frac{d\phi}{dr} - \frac{1}{r^2}(1-e^{-2\Lambda}) = 8\pi G A^4(\varphi)p + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 - \frac{1}{2}V(\varphi), \tag{3}$$

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\phi}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr} = 4\pi G\alpha(\varphi)A^4(\varphi)(\rho - 3p)e^{2\Lambda} + \frac{1}{4}\frac{dV(\varphi)}{d\varphi}e^{2\Lambda}, \tag{4}$$

$$\frac{dp}{dr} = -(\rho + p)\left(\frac{d\phi}{dr} + \alpha(\varphi)\frac{d\varphi}{dr}\right), \tag{5}$$

$$\frac{e^{\Phi-\Lambda}}{r^4}\frac{d}{dr}\left[e^{-(\Phi+\Lambda)}r^4\frac{d\bar{\omega}(r)}{dr}\right] = 16\pi G A^4(\varphi)(\rho+p)\bar{\omega}(r), \tag{6}$$

where we have defined

$$\alpha(\varphi) = \frac{d\ln A(\varphi)}{d\varphi} \quad \text{and} \quad \bar{\omega} = \Omega - \omega. \tag{7}$$

Here p and ρ are the pressure and the energy density in Jordan frame. They are connected to the Einstein frame ones via $p_* = A^4(\varphi)p$ and $\rho_* = A^4(\varphi)\rho$. The conformal factor and the potential for the R^2 gravity have the explicit form $A(\varphi) = e^{-\frac{\varphi}{\sqrt{3}}}$ and $V(\varphi) = \frac{1}{4a}\left(1 - e^{-\frac{2\varphi}{\sqrt{3}}}\right)^2$.

The boundary conditions are the natural ones to ensure regularity of the geometry at the center of the star, namely $\rho(0) = \rho_c, \Lambda(0) = 0, \frac{d\varphi(0)}{dr} = 0$ and $\frac{d\bar{\omega}(0)}{dr} = 0$, where the ρ_c is the central value for the energy density. The boundary conditions at infinity are $\lim_{r\to\infty}\phi(r) = 0, \lim_{r\to\infty}\bar{\omega}(r) = \Omega$ and $\lim_{r\to\infty}\varphi(r) = 0$ which ensures the asymptotic flatness.

Although the above equations are in the Einstein frame, the final results are presented in the physical frame. The physical radius of the star is given by $R_S = A[\varphi(r_S)]r_S$, where the coordinate radius r_S is defined by the condition $p(r_S) = 0$.

The moment of inertia I is defined in the standard way, $I = J/\Omega$, where J is the angular momentum of the star. The moment of inertia is calculated using the more convenient for numerical computations integral expression:

$$I = \frac{8\pi G}{3}\int_0^{r_S} A^4(\varphi)(\rho+p)e^{\Lambda-\Phi}r^4\left(\frac{\bar{\omega}}{\Omega}\right)dr. \tag{8}$$

The dimensionless parameter $a \to a/R_0^2$ and the dimensionless moment of inertia $I \to I/M_\odot R_0^2$, where $M_\odot$ is the solar mass and R_0 is one half the solar gravitational radius $R_0 = 1.47664\,\mathrm{km}$ are used.

2.2 Perturbation equations in the Cowling approximation

The perturbation equations for the non-radial oscillations of spherically symmetric compact stars in STT are presented in [3]. The so-called Cowling approximation is adopted, i. e. the fluid perturbations on a fixed Jordan frame metric are examined, which is equivalent to fixed scalar field and metric in the Einstein frame.

The explicit form of the system of equations we are solving is

$$\begin{aligned} \frac{dW}{dr} &= \frac{d\rho}{dp}\left[\omega^2 A(\varphi)e^{\Lambda-2\Phi}Vr + \Phi' W + \alpha(\varphi)\psi W\right] - \frac{l(l+1)A(\varphi)e^{\Lambda}V}{r} - (l+1)\frac{W}{r}, \\ \frac{dV}{dr} &= \left[2\left(\Phi' + \alpha(\varphi)\psi\right) - \frac{l}{r}\right]V - \frac{e^{\Lambda}WA^{-1}(\varphi)}{r}, \end{aligned} \tag{9}$$

with the following boundary condition at the surface of the star

$$\omega^2 e^{-2\Phi} rV + \left(\Phi' + \alpha(\varphi)\psi\right) e^{-\Lambda} W A^{-1} = 0\Big|_{r=R} \tag{10}$$

and at the center

$$W = -lA(\varphi)V|_{r=0}\,. \tag{11}$$

The above system (9), combined with the boundary conditions, forms an eigenvalue problem with the frequencies ω being the eigenvalues of the system.

3 Results

We investigate the mass to radius relations, the moment of inertia, and the f-mode oscillation frequencies in R^2 gravity and compare them to the GR case. The presented results are obtained numerically for hadronic and quark EOS [1–3]. Results for wide range of values for the free parameter a are presented. The maximal allowed by the observations dimensionless value for the the parameter is $a \sim 10^5$ or in dimensional units $a \leq 5 \times 10^{11}\mathrm{m}^2$. The deviation from GR for the maximal value of the parameter a marginally differs from the highest adopted by us dimensionless value, namely $a = 10^4$.

In Fig. 1 the mass to radius relations for two hadronic and one quark EOS are presented. The chosen EOS are with maximal masses around the observational limit of two solar masses. The observed behaviour for all realistic EOS is qualitatively similar – for larger masses the presence of a scalar field leads to an increase of the radius of the star, and for smaller masses, the radius decreases. However, the behaviour of the strange stars is different. For smaller masses, roughly below one solar mass, the deviations from the GR case is negligible. Bigger differences occur close to the maximal masses. Despite the different qualitative behaviour for the two kinds of EOS, in both cases the increase of the maximal mass is around 10%. The presented results for the mass and for the radius are obtained in the slow rotation approximation, but they coincide with the results in the static case, because the changes of M and R are of order Ω^2, and the approximation is linear in Ω.

The slow rotation approximation allows us to obtain a global characteristic of the neutron stars, namely the moment of inertia, given by equation (8). The moment of inertia of the neutron star as a function of the mass is plotted in Fig. 2 for different values of the parameters a. The maximal deviation from GR is for the maximal adopted value for the parameter a, and when we decrease the value of the parameter a, the values of the moment of inertia get closer to the GR, case and in the limiting case of $a \to 0$ they coincide. The maximal relative deviation for models with equal masses is about 30%. The maximal values of the moment of inertia for stable star models in R^2 gravity increases up to 40 % with respect to GR due to the increase in the maximal mass of the models.

In Fig. 3 we proceed to the investigation of the most common in the literature asteroseismology relations. The presented results are for a wider range of hadronic and quark EOS with different stiffnesses in order to test the EOS dependence of the results. Results for pure GR and R^2 gravity with $a = 10^4$ are presented in order to examine the maximal deviations of the R^2 gravity results from the GR ones. In the left panel of Fig. 3 the oscillation frequencies as a function of the mean density of the star are presented. The observed deviation in R^2 gravity with respect to the GR case is up to 10%. However, the results for the stiffest EOS, as well as these for the examined quark EOS deviates from the rest of the results. On the middle panel the normalised frequencies $M\omega$ as function of the compactness M/R are plotted. In this case the results in the two theories have similar qualitative behavior, but they are shifted with respect to one another and the difference is up to 5%. Also, the results for strange stars are shifted from the neutron stars ones. In the right panel of Fig. 3 the normalised frequencies as a function of the so-called effective compactness, $\eta \equiv \sqrt{M^3/I}$, are presented. Similar to the previous case, the results in both theories look qualitatively the same, but they are again shifted and in this case the difference between the results for neutrons and for strange stars is seriously reduced. The deviations from pure GR is up to 10%. The last two relations turn out to be not only EOS independent but are insensitive to the gravitational theory too.

4 Conclusions

We investigated non-perturbativly and self-consistently static and slowly rotating neutron and strange stars in GR and R^2 gravity. The rotating models we investigated in slow rotation approximation. In addition different asterosiesmologycal relations for the f-mode oscillation frequencies are studies for a wide range of hadronic and strange matter EOS with different stiffnesses. The perturbation equations were derived in the Cowling approximation.

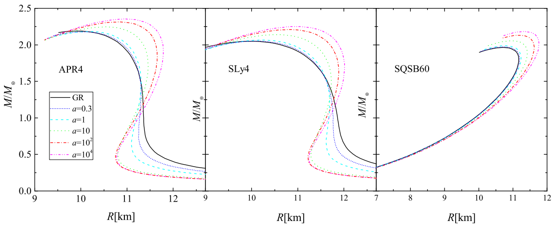

Figure 1. The mass to radius diagram for EOS APR4 (left panel), SLy4 (middle panel) and the strange star EOS (right panel).

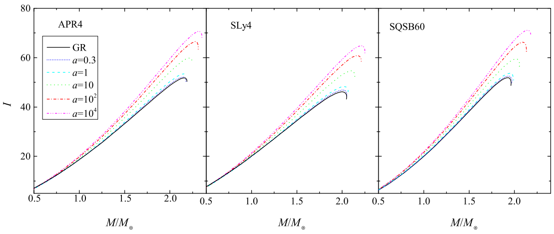

Figure 2. Moment of inertia as a function of mass for realistic equations of state – EOS APR4 in the left panel, EOS SLy4 in the middle panel and the strange star EOS in the right panel.

The observed deviation in the mass of radius relation due to the R^2 gravity is up to 10%. The increase of the moment of inertia, however, is much bigger – the deviation from GR for the maximal adopted value of the free parameter in the R^2 gravity for stable maximal mass models is up to 40%, which is beyond the EOS uncertainty for the EOS we have used. We investigated several popular in the literature gravitational wave asteroseismology relations between the oscillation frequency and the parameters of the star. These relation can serve as an astrophysical implications of our models. The results in GR and R^2 gravity are qualitatively the same in most of the cases – they are only shifted with respect to one another. The only exception is the relation connecting the f-mode oscillation frequencies to the average density of the star. In this case some qualitative differences between the two theories exist, but these differences do not exceed 10%. Therefore, the relations we consider turned out to be not only EOS independent, but up to a large extent theory independent too.

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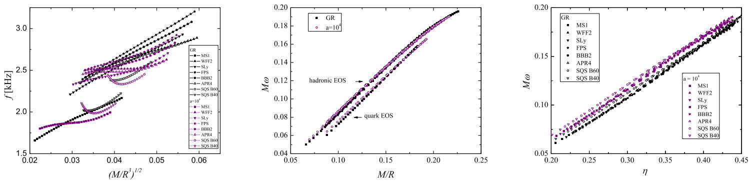

Figure 3. Different asteroseismology relations for neutron and strange stars. Results for GR and the maximal adopted value of the parameter a are presented.